Controllability and universal three-qubit quantum computation with trapped electron states

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We show how to control and perform universal three-qubit quantum computation with trapped electron quantum states. The three qubits are the electron spin, and the first two quantum states of the cyclotron and axial harmonic oscillators. We explicitly show how universal 3-qubit gates can be performed. As an example of a quantum algorithm, we outline the implementation of the 3-qubit Deutsch-Jozsa algorithm in this system.

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I. INTRODUCTION

Since the eighties, much effort has been put into the study of quantum computers, and various proposals for the physical implementation have been put forward in very different fields such as those of cold trapped ions [1], quantum dots [2], NMR [3] and neutral atoms [4]. Experimental implementations have, however, been few and limited to a low number of qubits. It is therefore interesting to study alternative candidates for quantum computers.

In the present work we study a scheme put forward for instance in [5] in which an electron trapped in a Penning trap is considered a possible realization of quantum logic. Advantages such as an almost complete absence of decoherence mechanisms and already obtained good experimental accuracy suggests this to be an interesting direction of study. We show that it is possible to perform universal three-qubit quantum computation in this system, and outline the implementation of the three-qubit Deutsch-Jozsa algorithm in this system.

In our scheme, quantum information is stored in an internal degree of freedom (spin) and two external degrees of freedom (cyclotron and axial motion) of the trapped electron. For two qubits, proposals have already been put forward in [5] and [6]. Here, instead of restricting ourselves to two qubits, we exploit an additional degree of freedom in a single electron and consider three qubits.

Note that although universal quantum computing for three qubits of a single electron has already been demonstrated theoretically in [7], this proposal relied on small relativistic effects which lead to anharmonicities for the cyclotron oscillator. In our scheme, we treat the cyclotron oscillator as essentially harmonic (to experimental resolution) and we use a traveling field that is experimentally well-established [8]. This scheme is distinctly different from schemes in alternative trap configurations such as in Refs. [9] and [10], where a linear array of Penning traps was investigated, and in [11], where an array of planar Penning traps was considered.

In the following section, we briefly describe the physical system and the theoretical model that describes it. Then we describe the control mechanisms and show the control equations. Section 4 demonstrates that universal 3-qubit quantum computation is possible using these controls. Subsequently, we present an explicit example of the 3-qubit Deutsch-Jozsa algorithm that can be implemented. In the last section, we discuss the experimental challenges to the implementation of this scheme.

II. THE SYSTEM

The physical system consisting of a single electron trapped in a Penning trap has already been described in some detail in for instance [12]. To summarize, the electron experiences a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ and a static quadrupole potential

$$V = V_0 \frac{z^2 - (x^2+y^2)/2}{2d^2}$$

where V_0 is the potential between the trap electrodes and d is a characteristic length of the trap [13]. The Hamiltonian, given by

$$H_0 = \frac{1}{2m}\left(\mathbf{p} - e\mathbf{A}_0\right)^2 + eV - \boldsymbol{\mu}\cdot\mathbf{B}$$

where $\mathbf{A}_0 = \frac{1}{2}\mathbf{B}\times\mathbf{r}$ and $\boldsymbol{\mu} = \frac{ge\hbar}{4m}\boldsymbol{\sigma}$, can be recast in terms of three harmonic oscillators [13]

$$H_0 = \hbar\omega_c' a_c^\dagger a_c + \hbar\omega_z a_z^\dagger a_z - \hbar\omega_m a_m^\dagger a_m + \frac{\hbar}{2}\omega_s\sigma_z$$

namely the cyclotron, axial and magnetron motions. The frequencies are given by $\omega_c' = (\omega_c + \tilde{\omega}_c)/2$, $\omega_m = (\omega_c - \tilde{\omega}_c)/2$, $\omega_z = \sqrt{\frac{eV_0}{md^2}}$, $\tilde{\omega}_c = \sqrt{\omega_c^2 - 2\omega_z^2}$, $\omega_c = \frac{|e|B}{m}$ and $\omega_s = \frac{g|e|B}{2m}$. In [5] the spin and axial degrees of freedom are used as qubits whereas in [6] spin and the

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cyclotron oscillator constitute the qubits. Here we use the spin of the electron and the axial and the cyclotron oscillator states for the three qubits. The logical states $|0\rangle$ and $|1\rangle$ corresponds to $|\downarrow\rangle$ and $|\uparrow\rangle$ for the spin and the Fock states $|0\rangle$ and $|1\rangle$ for the cyclotron and the axial oscillators. The states are denoted by $|jnl\rangle$ where j, n, l are the logical states for the spin, cyclotron and axial states, respectively. The frequencies corresponding to the qubit-energy splittings are of the order GHz, GHz and MHz, respectively. See Fig. 1 for an energy-level diagram.

III. CONTROLLING THE SYSTEM

The spin qubit is the only qubit which can be controlled directly. This is done using a small transverse magnetic field [12]

$$\mathbf{b}(t) = b\left(\cos(\omega t + \phi)\hat{x} + \sin(\omega t + \phi)\hat{y}\right)$$

If ω is close to ω_s this leads to the following Hamiltonian in the interaction picture

$$H'_s = \frac{\hbar\Omega}{2}\left(\sigma_+ \exp(-i\phi) + \sigma_- \exp(i\phi)\right)$$

where $\Omega = \frac{g|e|b}{2m}$. In the basis $|\downarrow nl\rangle$, $|\uparrow nl\rangle$ this interaction implies the evolution matrix

$$U'_s = \begin{pmatrix} \cos(\frac{\Omega t}{2}) & -ie^{i\phi}\sin(\frac{\Omega t}{2}) \\ -ie^{-i\phi}\sin(\frac{\Omega t}{2}) & \cos(\frac{\Omega t}{2}) \end{pmatrix}$$

subsequently referred to as a $p_s(\theta, \phi)$ pulse, where $\theta = \Omega t$.

The cyclotron and axial qubits cannot be directly controlled. Cyclotron transitions can, admittedly, be addressed by setting up a proper vector potential [12], but since the levels are equally spaced, population will inevitably leave the computational subspace [14]. To control the spin-axial transition a traveling magnetic field is set up

$$\mathbf{b}(t) = b\left(\cos(\omega t + \phi - kz)\hat{x} + \sin(\omega t + \phi - kz)\hat{y}\right)$$

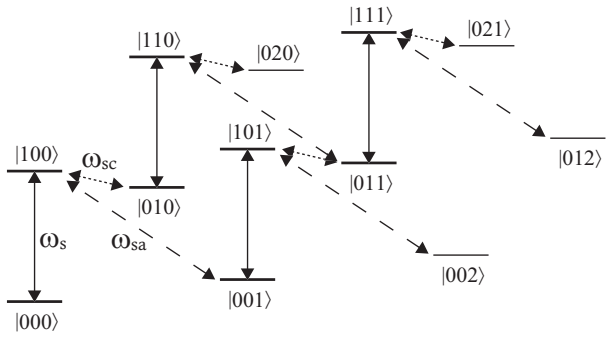

FIG. 1: Energy-level diagram. States are denoted $|jnl\rangle$, where j is the spin state, n the cyclotron state and l the axial state. Levels marked with bold are included in the computational subspace. Solid, dashed and dotted lines indicate spin, spin-axial and spin-cyclotron transitions, respectively.

which leads to the following Hamiltonian in the interaction picture

$$H'_{sa} = \frac{\hbar\Omega}{2}\big(\sigma_+ \exp(i(\eta(a_z^\dagger e^{i\omega_z t} + a_z e^{-i\omega_z t}) - \Delta\cdot t - \phi)) + \sigma_- \exp(-i(\eta(a_z^\dagger e^{i\omega_z t} + a_z e^{-i\omega_z t}) - \Delta\cdot t - \phi))\big)$$

where $\Omega = \frac{g|e|b}{2m}$, $\Delta = \omega - \omega_s$ and $\eta = k\sqrt{\frac{\hbar}{2m\omega_z}}$. For $\omega = \omega_{sa} = \omega_s - \omega_z$ only the levels $|\downarrow n(l+1)\rangle$ and $|\uparrow nl\rangle$ are connected. In the basis $|\downarrow n(l+1)\rangle$, $|\uparrow nl\rangle$ the evolution matrix is given by

$$U'_{sa} = \begin{pmatrix} \cos(\frac{\theta_l}{2}) & -e^{i\phi}\sin(\frac{\theta_l}{2}) \\ e^{-i\phi}\sin(\frac{\theta_l}{2}) & \cos(\frac{\theta_l}{2}) \end{pmatrix} \quad (1)$$

where $\theta_l = \Omega t\cdot\eta e^{-\eta^2/2}\sqrt{\frac{1}{l+1}}\cdot L_l^1(\eta^2)$. $L_n^m(x)$ is a generalized Laguerre polynomial. In the following this evolution matrix will be referred to as a $p_{sa}(\theta, \phi)$ pulse. Note that this does not assume the Lamb-Dicke limit, so the extent of the zero-point motion of the harmonic oscillator can be a significant fraction of the wavelength of the field.

A spin-cyclotron interaction can be set up using a magnetic field as described in [6]

$$\mathbf{b}(t) = b(x\hat{x} + y\hat{y})\cos(\omega t + \phi)$$

This leads to the Hamiltonian

$$H'_{sc} = \frac{g\mu_B b}{2}\sqrt{\frac{\hbar}{2m\tilde{\omega}_c}}(\sigma_+ a_c e^{-i\phi} + \sigma_- a_c^\dagger e^{i\phi})$$

If $\omega = \omega_{sc} = \omega_s - \omega'_c$ only the levels $|\downarrow nl\rangle$, $|\uparrow(n-1)l\rangle$ are connected. In the basis $|\downarrow nl\rangle$, $|\uparrow(n-1)l\rangle$ the following evolution operator is thus obtained

$$U'_{sc} = \begin{pmatrix} \cos(\frac{\sqrt{n}\theta}{2}) & ie^{i\phi}\sin(\frac{\sqrt{n}\theta}{2}) \\ ie^{-i\phi}\sin(\frac{\sqrt{n}\theta}{2}) & \cos(\frac{\sqrt{n}\theta}{2}) \end{pmatrix} \quad (2)$$

where $\theta = -g\mu_B b\sqrt{\frac{1}{2m\hbar\tilde{\omega}_c}}t$. In the following this evolution matrix will be referred to as a $p_{sc}(\theta, \phi)$ pulse.

Fig. 1 shows that the three fields connect all the eigenstates of the spin-axial-cyclotron system. In addition, the levels are connected in such a way that the system is eigenstate controllable [15]. That is, population can be coherently transferred from any eigenstate to any other eigenstate. For example, consider the set of eigenstates illustrated in Fig. 1. The condition for eigenstate controllability is that the pulses of frequency ω_{sa}, ω_s and ω_{sc} must be applied *sequentially*, and not simultaneously. For example, let us say we want to transfer the $|000\rangle$ state to the $|111\rangle$ state. We can do so by the sequence: $p_s(\pi, \phi_1)$, $p_{sa}(\pi, \phi_2)$, $p_s(\pi, \phi_3)$, $p_{sc}(\pi, \phi_4)$, $p_s(\pi, \phi_5)$, where ϕ_1, ϕ_2, ϕ_3, ϕ_4 and ϕ_5 are arbitrary phases. This is similar in spirit to the eigenstate transfer schemes for trapped ions [16].

IV. UNIVERSAL QUANTUM COMPUTATION

A universal set of gates consists of the Hadamard, the T-gate and the controlled-NOT gates [17]:

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}, H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

In the following, we will demonstrate how to implement a universal set of gates for the current system by applying the interactions described above.

For the spin qubit, the single-qubit gates are easily implemented. Thus the T gate is performed up to a global phase factor using the two pulses $p_s(\pi, \pi/8)$, $p_s(\pi, 0)$. An arbitrary phase gate $\left(\begin{smallmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{smallmatrix}\right)$ is in fact performed just by $p_s(\pi, \varphi/2)$, $p_s(\pi, 0)$. The Hadamard gate is obtained from the pulse sequence $p_s(\pi, -\pi)$, $p_s(\pi/2, \pi/2)$.

Implementing a swap gate

$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

between the spin and cyclotron qubits is complicated by the fact that driving the transitions $|01l\rangle \leftrightarrow |10l\rangle$ simultaneously drives the transitions $|02l\rangle \leftrightarrow |11l\rangle$ and population thus leaves the computational subspace. This problem can, however, be overcome using composite pulses [18], a method that has already been applied to for instance trapped ions [19] and trapped electrons [6]. Applying the pulse sequence $p_{sc}(\pi/\sqrt{2}, 0)$, $p_{sc}(2\pi/\sqrt{2}, \phi_s)$, $p_{sc}(\pi/\sqrt{2}, 0)$, where $\phi_s = \arccos(\cot^2(\pi/\sqrt{2}))$, implements the swapping gate up to a phase factor of $e^{i\alpha}$ for $|01\rangle$ and $-e^{-i\alpha}$ for $|10\rangle$, where $\alpha \approx -0.8652$. To convert it to a proper swap gate we suggest the following. First, $p_s(\pi, \alpha/2)$, $p_s(\pi, 0)$ followed by the swapping gate and subsequently the pulses $p_s(\pi, (\pi - \alpha)/2)$, $p_s(\pi, 0)$ implements a swap gate up to a phase of π for $|00\rangle$. As demonstrated in [6] the pulse sequence $p_{sc}(\pi, 0)$, $p_{sc}(\pi/\sqrt{2}, \pi/2)$, $p_{sc}(\pi, 0)$, $p_{sc}(\pi/\sqrt{2}, \pi/2)$ leads to a controlled-phase gate endowing all states but $|00\rangle$ with a phase of π. Thus the phase of $|00\rangle$ is corrected by applying this controlled-phase gate.

Now implementing any single-qubit gate for the cyclotron qubit is easy, since $I \times S = SWAP \cdot (S \times I) \cdot SWAP$, where S denotes an arbitrary single-qubit gate. Implementing for instance a T gate, however, simplifies since sandwiching a T gate for the spin qubit between the swapping sequence and the same sequence with all phases offset by π leads to a T gate for the cyclotron qubit.

To implement a controlled-NOT gate between the spin and the cyclotron qubit, we first implement the controlled-phase gate as before by the pulse sequence $p_{sc}(\pi, 0)$, $p_{sc}(\pi/\sqrt{2}, \pi/2)$, $p_{sc}(\pi, 0)$, $p_{sc}(\pi/\sqrt{2}, \pi/2)$. To convert it to a standard CZ gate

$$CZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

we simply sandwich it between NOT gates on both qubits. Finally, applying a Hadamard gate to the cyclotron qubit before and after the CZ gate leads to a $CNOT$ gate.

To implement a swap gate between the spin and the axial qubits p_{sa} pulses are naturally employed. As before, it is a problem that driving for instance the transitions $|0n1\rangle \leftrightarrow |1n0\rangle$ simultaneously drives the transitions $|0n2\rangle \leftrightarrow |1n1\rangle$ and population thus leaves the computational subspace. Noting that $\theta_{l=1} = \frac{1}{\sqrt{2}}(2 - \eta^2)\theta_{l=0}$ we can, however, in the limit where $\eta \approx 0$ (for which $\theta_{l=1} = \sqrt{2}\theta_{l=0}$) use the composite pulse idea employed for the spin-cyclotron interaction. If all phases are offset by $-\pi/2$ (cf. formulas (1) and (2)) we can use exactly the same sequences as for the spin-cyclotron case to implement the desired gates. Notice that this method can also be applied for a nonzero η. For instance, choosing $\eta = 2$ leads to $\theta_{l=1} = -\sqrt{2}\theta_{l=0}$ and the pulses above are immediately applicable.

A controlled-NOT gate between the cyclotron and the axial qubits is implemented by the sequence $CNOT_{cyclo,spin}$, $CNOT_{spin,axial}$, $CNOT_{cyclo,spin}$, $CNOT_{spin,axial}$.

So we have now demonstrated how to perform universal computation with three-qubits in a trapped electron scheme.

V. IMPLEMENTING THE DEUTSCH-JOZSA ALGORITHM

In this section, we will demonstrate how to implement the 3-qubit Deutsch-Jozsa (DJ) algorithm [20]. The refined version suggested in [21] allows us to test functions for three qubits instead of just two with three physical qubits available, as the control register is eliminated. Testing functions on three qubits compared to two is interesting, since three is the minimum number of qubits required to solve the DJ algorithm in a non-classical way [21]. The algorithm works in the following way: 1) Initialize the system to $|000\rangle$, 2) Apply a Hadamard gate to each qubit, 3) Apply the unitary operator U_f defined by $U_f|x\rangle = (-1)^{f(x)}|x\rangle$, 4) Apply a Hadamard gate to each qubit, 5) Measure the state of the qubits. If the measurement returns $|000\rangle$ the function is constant, else it is balanced.

We have already demonstrated how to implement a Hadamard gate for each qubit, so we only need to implement the U_f gates. Now, for three qubits there are 2 constant functions and 70 balanced functions. These

functions are indexed as in [22] using their outputs expressed as hexadecimal numbers such that for instance the function with the output $f(0)\cdots f(7) = 00001111$ is denoted f_{0F}. Since $U_{f_x} = -U_{f_{FF-x}}$ we only need to implement 1 constant and 35 balanced functions. The constant function corresponding to f_{00} is easily implemented since $U_{f_{00}}$ is equal to the identity. To implement the balanced functions we refer to [22], where NMR sequences for the 35 balanced functions are written down. From this work it follows that all functions can be constructed from gates on the form

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & \pm i \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \pm i & 0 & 0 \\ 0 & 0 & \pm i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The single-qubit gates are easily implementable for the spin qubit. The first one, corresponding to $I_z(\pi)$ in [22], is obtained from the sequence $p_s(\pi, \pi/2)$, $p_s(\pi, 0)$, whereas the second one, corresponding to $I_z(\pm\pi/2)$, is given by $p_s(\pi, \pm\pi/4)$, $p_s(\pi, 0)$. Using $SWAP$ gates these gates are transformed to single-qubit gates for the cyclotron and the axial qubits and thus all necessary single-qubit gates listed in [22] are obtained. Likewise the two-qubit operations are easily achieved. Thus the sequence $CNOT, \left(\begin{smallmatrix} 1 & 0 \\ 0 & \pm i \end{smallmatrix}\right), CNOT$ implements the desired two-qubit gates, that is, the gates $J_{ij}(\pm\pi/2)$ in [22]. It follows that all the U_f functions are implementable, and therefore that the Deutsch-Jozsa algorithm can be executed in our system.

VI. DISCUSSION AND CONCLUSION

In conclusion, we have demonstrated universal quantum computation for trapped electron three-qubit states. We have explicitly determined a universal set of gates by finding the necessary pulse sequences. We have also demonstrated that the three-qubit Deutsch-Jozsa algorithm can be implemented in this system.

Thus, this work expands current work in the area of trapped-electron quantum computing to exploit another degree of freedom in each electron. Besides constituting an interesting system in itself in that it can be used to test for example the Deutsch-Jozsa algorithm, this work could also be useful for studying many-particle gates via the Coulomb interaction between electrons in planar Penning traps (the system is not scalable with the trap considered in this proposal). Each particle could have three qubit degrees of freedom and one of these could be used for communication. Such a situation is highly interesting for the case of error correction.

However, there are significant experimental challenges to the implementation of this proposal at present. The primary challenge is that the axial motion cannot yet be cooled to the ground state (which is necessary for initialization). There are proposals to do so using the cyclotron motion, but these are still in their early stages. Nevertheless we expect that our work will motivate rapid experimental progress in this direction, much as early quantum computing proposals [1] motivated the motional cooling of trapped *ions* [23] about a decade ago. A second challenge is that all the three-qubit states cannot be measured directly. The spin qubit can be measured [8], as can the axial qubit with a low temperature circuit [24]. For the cyclotron qubit, a scheme adopted in [12] can be used. In that work, in which the spin and cyclotron levels are qubits, it is demonstrated that the spin and cyclotron levels can be coupled with the axial motion and subsequently the axial motion can be measured to get a cyclotron qubit measurement. Although this system has long coherence times, there are notable sources of decoherence that set a limit on the number of gate operations that can be performed, for instance fluctuating magnetic and electric fields, thermal noise and intensity noise from the microwave sources. In spite of these experimental challenges to be overcome, this system provides an attractive quantum computing paradigm to be investigated.

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